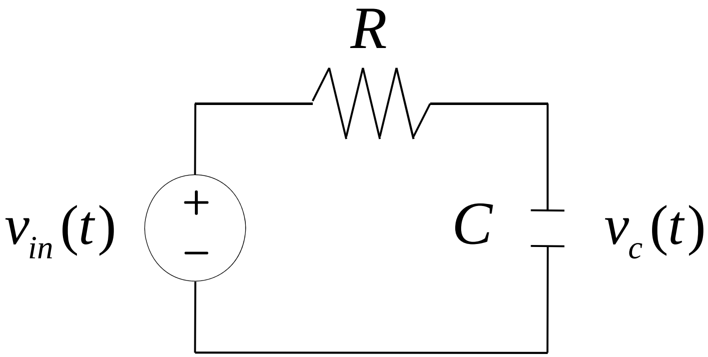
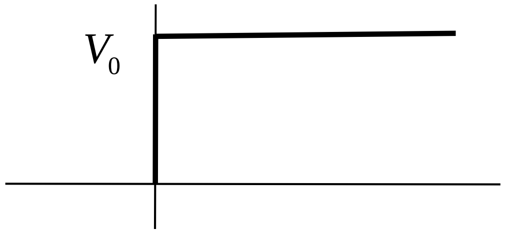
# Lect. 3: Review of Basic Circuits (2)




When $v_{in}(t)$




$v_c(t) = ?$

- Initially, capacitor has no charges, $v_c(t) = 0$ $(Q = CV)$
- Current flows from $v_{in}$ to $C$ through $R$
- Charges build up and $v_c(t)\uparrow$ $\left(i = C\dfrac{dv}{dt}\right)$
- Current flow gets smaller
- Eventually no current flows, $v_c(t) = V_0$

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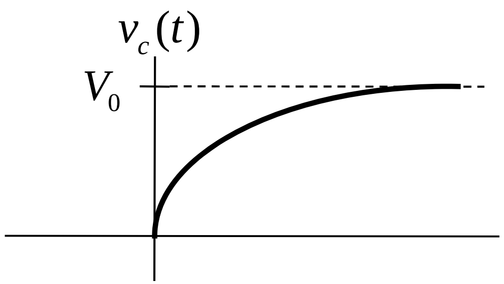
$$v_{in}(t) - i(t)R = v_c(t)$$

$$i(t) = C\frac{dv_c(t)}{dt}$$

$$v_{in}(t) - RC\frac{dv_c(t)}{dt} = v_c(t) \qquad \therefore \frac{dv_c(t)}{dt} + \frac{v_c(t)}{RC} = \frac{v_{in}(t)}{RC}$$ ;1st-order differential eq.




$$v_c(t) = V_0(1 - e^{-t/\tau})$$

$t = 0 \qquad v_c(t) = 0$

$t = \infty \qquad v_c(t) = V_0$

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Special case: sinusoidal steady-state

Assume $v_{in}(t) = V_{in} e^{j\omega t}$

In linear circuit, frequency does not change.

Only magnitude and phase change.

$$\therefore v_c(t) = V_c e^{j\omega t}$$

($V_{in}$, $V_c$: phasors)

Substituting above into the governing eq.

$$j\omega V_c e^{j\omega t} + \frac{V_c e^{j\omega t}}{RC} = \frac{V_{in} e^{j\omega t}}{RC} \qquad V_c(j\omega + \frac{1}{RC}) = \frac{V_{in}}{RC}$$

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$$V_c = V_{in} \cdot \frac{1}{RC} \cdot \frac{1}{j\omega + \frac{1}{RC}} = V_{in} \cdot \frac{1}{j\omega RC + 1}$$

$$\therefore v_c(t) = V_{in} \cdot \frac{1}{j\omega RC + 1} \cdot e^{j\omega t} = \frac{1}{j\omega RC + 1} \cdot v_{in}(t)$$

➔ $v_c(t)$ experiences magnitude and phase change from $v_{in}(t)$

Our interest: $\left|\frac{1}{j\omega RC + 1}\right|, \ \angle \frac{1}{j\omega RC + 1}$

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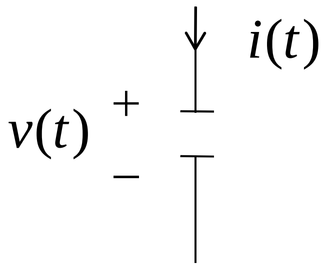
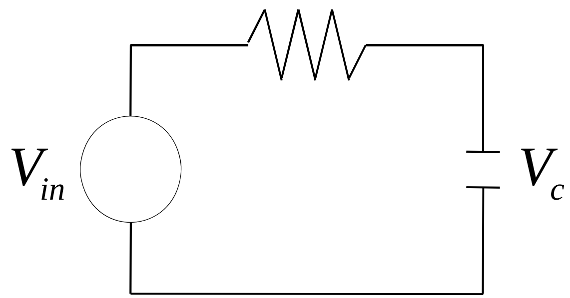
In sinusoidal steady-state,
we can define impedance, $V(\omega)/I(\omega)$, for any circuit element




$$v(t) = Ve^{j\omega t}$$

$$i(t) = Ie^{j\omega t} = C\frac{dv(t)}{dt} = Cj\omega Ve^{j\omega t}$$

$$\therefore \frac{V}{I} = Z(\omega) = \frac{1}{j\omega C}$$




Using voltage divider,

$$\frac{V_c}{V_{in}} = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} = \frac{1}{1 + j\omega RC}$$

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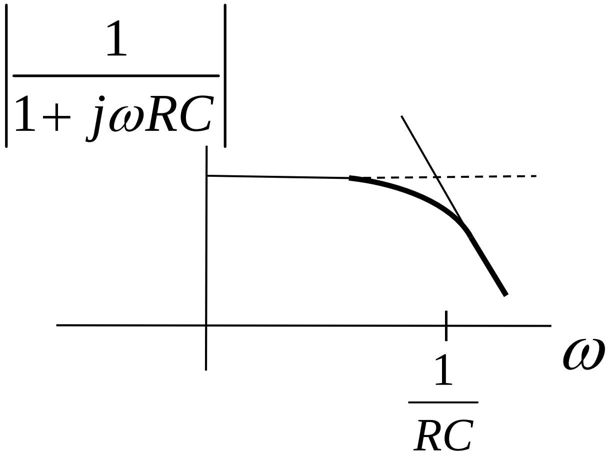
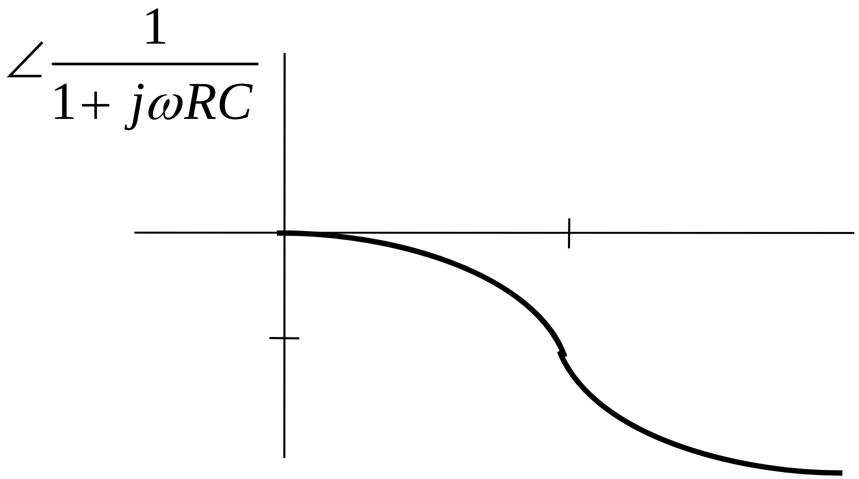



$$\omega << 1 \qquad | \, | \sim 1$$

$$\omega >> 1 \qquad | \, | \sim \frac{1}{\omega RC}$$




$$\omega << 1 \qquad \angle \sim 0$$

$$\omega = \frac{1}{RC} \qquad \frac{V_c}{V_{in}} = \frac{1}{j+1} \qquad \angle = -45^\circ$$

$$\omega >> 1 \qquad \frac{V_c}{V_{in}} \sim \frac{1}{j\omega} \qquad \angle \sim -90^\circ$$

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Homework: (1) Determine $v_L(t)$ in the RL circuit below. $v_{in}(t)$ is shown below.

(2) Plot $\left|\dfrac{V_L(\omega)}{V_{in}(\omega)}\right|$ and $\angle\left|\dfrac{V_L(\omega)}{V_{in}(\omega)}\right|$.